

MULTI-DECOMPOSITION OF COMPLETE TRIPARTITE GRAPHS INTO TREES OF ORDER FIVE

Hemalatha P¹ and Chaadhanaa A²

^{1,2}Department of Mathematics, Vellalar College for Women,
Erode - 638 012, Tamil Nadu, India
e-mail: dr.hemalatha@gmail.com¹, math.chaad@gmail.com²

Abstract

A $\{P_5^\alpha, S_5^\beta, Y_5^\gamma\}$ -decomposition of a graph is a partition of its edge set into α copies of P_5 , β copies of S_5 , and γ copies of Y_5 , where P_5 , S_5 , and Y_5 are trees of order five. In this paper, we study such decompositions of the complete tripartite graph $K_{\ell,m,n}$ with $\ell \leq m \leq n$. We establish necessary and sufficient conditions for their existence. The constructions are obtained using Latin squares, which provide a systematic method for partitioning the edge set of $K_{\ell,m,n}$ into the prescribed tree structures. This work extends earlier cycle-based decomposition results to non-isomorphic trees of fixed order.

Keywords: Paths, Stars, Y -Trees, Complete tripartite graphs, Multi-decomposition, Latin Square

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